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River Crossing 1 – Farm Produce

Alcuin of Northumbria, aka Flaccus Albinus Alcuinus or Ealhwine, was a scholar, a clergyman and a poet. He lived in the eighth century and rose to be a leading figure at the court of the emperor Charlemagne. He included this puzzle in a letter to the emperor, as an example of 'subtlety in Arithmetick, for your enjoyment'. It still has mathematical significance, as I'll eventually explain. It goes like this.

A farmer is taking a wolf, a goat and a basket of cabbages to market, and he comes to a river where there is a small boat. He can fit only one item of the three into the boat with him at any time. He can't leave the wolf with the goat, or the goat with the cabbages, for reasons that should be obvious. Fortunately the wolf detests cabbage. How does the farmer transport all three items across the river?

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There are two solutions. One is:

(1) Take the goat across.

(2) Come back with no cargo, pick up the wolf, and take that across.

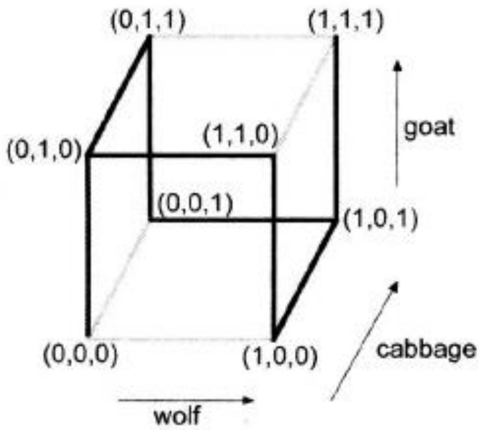
(3) Bring the goat back, but leave the wolf.

(4) Drop off the goat, pick up the cabbage, cross the river, leave the cabbage.

(5) Come back with no cargo, pick up the goat, take it across.

In the other, the roles of wolf and cabbage are exchanged.

I like to solve this geometrically, using a picture in *wolf–goat–cabbage space*. This consists of triples (w, g, c) where each symbol is either 0 (on this side of the river) or 1 (on the far side). So, for instance, $(1, 0, 1)$ means that the wolf and cabbage are on the far side but the goat is on this side. The problem is to get from $(0, 0, 0)$ to $(1, 1, 1)$ without anything being eaten. We don't need to say where the farmer is, since he always travels in the boat during river crossings.



Wolf–goat–cabbage space: now it's obvious.

There are eight possible triples, and they can be thought of as the vertices of a cube. Because only one item can accompany the farmer on each trip, the permissible moves are the edges of the cube.

However, four edges (shown in grey) are not permitted, because things get eaten. The remaining edges (black) do not cause mayhem.

So the puzzle reduces to a geometric one: find a route along the black edges, from $(0, 0, 0)$ to $(1, 1, 1)$. The two solutions are immediately evident.